Waves & Oscillations:

★ **Periodic motion:** The motion which repeats itself after regular intervals of time is known as periodic motion.

Ex: Swinging of a pendulum.
Rotation of the Earth.

★ **Time period:** The fixed time after which periodic motion repeats itself is known as time period. (T).

★ **Frequency:** The reciprocal of time period is known as frequency(f).

$$\boxed{f = \frac{1}{T}}$$

Unit: Hertz.

★ **Angular frequency (ω):** The number of cycles per unit time is known as angular frequency (ω).

Mathematically: $T = \dfrac{2\pi}{\omega}$. or $\omega = \dfrac{2\pi}{T}$.

★ **Simple Harmonic Motion (SHM):** It is a type of periodic motion in which the restoring force is directly proportional to the force applied.

Mathematically: $F \propto -x$, where the -ve sign indicates that the force and displacement are opposite.

$\Rightarrow F = -kx$, where k is the const of proportionality called spring constant.

Ex: Swinging of a pendulum bob.

Mass attached to a spring and so on.

Equation of SHM: Any particle executing SHM is usually represented by the ~~fo~~ equation:

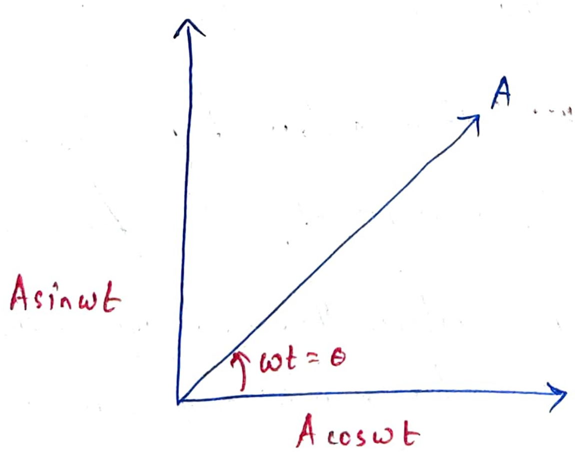
$Y = A \sin \omega t$ or $Y = A \cos \omega t$, where A is the amplitude.

where $\theta = \omega t$ is the angular displacement.

If we also consider the phase difference ϕ, then the equation is given by: $Y = A \sin(\omega t + \phi)$ or $Y = A \cos(\omega t + \phi)$

Phase difference (ϕ): Phase difference is the difference in phase angle between two oscillating bodies which have the same frequency, or it the angle by which a wave leads or lags the other wave.

Graphical representation of SHM:



Amplitude: The maximum displacement from the mean position is known as the amplitude (A).

Unit: metres.

Velocity of SHM: The velocity with which a simple harmonic wave moves.. It is the rate of change of displacement.

Now: $y = A\sin\omega t$

$$\frac{dy}{dt} = \frac{d}{dt}(A\sin\omega t)$$

$$\Rightarrow v = A\frac{d}{dt}\sin(\omega t)$$

$$\Rightarrow \boxed{v = A\omega\cos\omega t}$$

Velocity v is maximum when $\cos\omega t = 1$

$$\Rightarrow \cos\omega t = \cos 0$$

$$\Rightarrow \omega t = 0$$

$$\Rightarrow t = 0.$$

Velocity v is minimum when $\cos\omega t = 0$

$$\Rightarrow \cos\omega t = \cos \pi/2$$

$$\Rightarrow \omega t = \pi/2$$

$$\Rightarrow \frac{2\pi}{T} \cdot t = \pi/2$$

$$\Rightarrow t = T/4.$$

Acceleration of SHM wave: It is the time rate of change of velocity.

Now: Acceleration $a = \frac{dv}{dt}$

$= \frac{d}{dt} A\omega \cos\omega t$

$= A\omega \frac{d}{dt} \cos\omega t$

$= A\omega(\omega)(-\sin\omega t)$

$= -A\omega^2 \sin\omega t$

$= -\omega^2 (A\sin\omega t)$

$= -\omega^2 y$

Simple pendulum:

A simple pendulum consists of a small, heavy bob suspended by a light, inextensible string, fixed at one end and free to swing in a vertical plane under gravity.

The time period of a simple pendulum is given by:

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

where T is the time period. g is the acceleration due to gravity.
l is the length

Now angular frequency (ω):

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \boxed{\omega = \sqrt{g/l}}$$

Frequency $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{l}{g}}}$

Second pendulum:

A second's pendulum is a simple pendulum with a time period of 2 seconds (1 second in each direction)

We know: $T = 2\pi\sqrt{\frac{l}{g}}$

$$\Rightarrow \cancel{2} = \cancel{2}\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \frac{1}{\pi} = \sqrt{l/g}$$

$$\Rightarrow \frac{1}{\pi^2} = \frac{l}{g}$$

$$\Rightarrow \frac{g}{\pi^2} = l$$

$\therefore l = 0.994\,m$

$\approx 1m.$

So, a second's pendulum is approximately 1 m long.

Free vibrations:

When a system is set into motion and allowed to vibrate without any external force acting continuously on it, it is called a free vibrations.

The system vibrates at its natural frequency.

Forced vibrations:

When a system is subjected to a continuous external periodic force, it is known as forced vibrations. The system vibrates at the frequency of the external force; not its natural frequency.

Resonance: Resonance occurs when the frequency of the external force equals the natural frequency of the system. It results in a maximum amplitude of vibration.

At resonance: $f_{external} = f_{natural}$.

Wave-motion:

Wave-motion is a disturbance that travels through a medium (or space) carrying energy from one point to another without net transport of matter.

Characteristics of Wave Motion:

1) Crest: Highest point of a transverse wave.

2) Trough: Lowest point of a transverse ~~p.~~ wave.

3) Compression: Region of high pressure in a longitudinal wave.

4) Rarefaction: Region of low pressure in a longitudinal wave.

5) Amplitude (A): Maximum displacement from the mean position.

6) Wavelength (λ): Distance between two consecutive crests or compressions.

7) Frequency (f): Number of oscillations per second (Hz).

8) Time period (T): Time taken for one complete cycle.

9) Wave speed ($\bar{\lambda}$): Speed at which the wave travels.

$$\boxed{v = f\lambda}$$

Types of wave motion:

1) Transverse waves: ~~For~~ The waves in which particles of the medium vibrate perpendicularly to the direction of wave propagation.

Ex: Light wave, water wave.

2) Longitudinal waves: The waves in which particles of the medium vibrate parallel to the direction of wave propagation.

Ex: Sound wave, vibrations in a spring.

Stationary wave (standing wave):

A stationary wave is formed by the superposition of two identical progressive waves (same amplitude and frequency) travelling in opposite directions in the same medium.

As a result, the wave appears to stand still, with nodes (no displacement) and antinodes (maximum displacement).

Properties of Stationary wave:

1) There is no net energy transfer.
2) There are points of zero displacement called nodes.
3) There are points of maximum displacement called anti-nodes.
4) Different particles oscillate with different amplitudes.
5) The distance between two consecutive nodes or anti-nodes is $\frac{\lambda}{2}$.

Difference between progressive and stationary waves:

- In progressive wave, there is a transfer of energy in the medium and in stationary wave there is no energy transfer.
- In progressive wave the movement is forward in one direction and in stationary wave there is no forward movement.
- In progressive wave, the amplitude is same for all particles, whereas in ~~other~~ stationary wave the amplitudes varies from zero to max at anti-nodes.

4) In progressive wave, node & anti-nodes are present whereas in stationary waves, they are not present.

Propagation of sound:

Sound is a longitudinal wave that propagates through a medium by compression and rarefactions of particles.

In air sound travels due to successive pressure variations that move parallel to the direction of wave propagation.

It requires a material medium and cannot travel through vacuum.

Newton's formula for the velocity of sound in air:

Newton assumed that the sound wave propagation is an isothermal process and is given by:

$$\boxed{v = \sqrt{\frac{P}{\rho}}}$$

v is the velocity of sound.

P is the Pressure of air.

ρ is the density of air.

Laplace correction:

Laplace corrected the Newton's formula by considering that sound-wave propagation is an adiabatic process.

It is given by:

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}}$$

γ: Adiabatic constant.

This formula gives velocity of sound to be $v = 331\,ms^{-1}$ which is quite close to experimental value.

Factors affecting velocity of sound in air:

1) Temperature: Sound travels faster at higher temp.

$$v \propto \sqrt{T}$$

2) Humidity: More humid air has low density and hence sound travels faster.

3) Pressure: At constant temp and composition pressure has no effect.

4) Density of air: ~~Light~~ Sound travels slower if density of air is high.

5) Wind direction: If wind is in the direction of sound, speed increases, otherwise it decreases.

Audible range:

The range of sound frequecies that the human ear can hear is known as audible range.

Its frequency range is 20 Hz to 20,000 Hz.

* When frequency is below 20 Hz it is not audible (infrasonic)

  When frequency is above 20 kHz, it is not audible (ultrasonic)

Application of ultransonic wave to calculate depth:

It is based on echo sounding. If v is the velocity of the ultrasonic wave and t is the time taken to touch the bottom or ocean floor then the depth is given by:

$$\boxed{\text{Depth} = \frac{v \times t}{2}}$$

Reflection of sound: When sound wave hits a surface and bounces back, it is called reflection of sound.

Ex: Echoing of soun ~~d~~ from a solid wall.

Echo: A repitition of sound caused by the reflection from a distant surface is known as echo.

Condition to Hear an Echo: The reflected sound must reach the ear after 0.1 seconds.

The minimum distance of the reflector should be :

$$\text{Distance } d = \frac{v \times t}{2} = \frac{343 \times 0.1}{2} \approx 17.51 \text{ m.}$$

So, the reflector must be atleast 17.2 m. away.

Reverberation :

The prolonged persistence of sound in a space due to multiple reflections after the source has stopped is known as reverberation.

Reverberation time : The time taken for the sound to decay by 60 dB after the source has stopped is known as reverberation time.

Sabine's Law :

$$T = \frac{0.161 \times V}{A}$$

T : reverberation time (in seconds)

V : volume of the hall (in m^3)

A : Total absorption in the room (in sabins)

Acoustic Requirements of a Good Auditorium :

To ensure clear sound, an auditorium should have :

- Proper reverberation time.
- Sound absorbing materials like carpets, curtains etc.

- No echoes or dead spots.
- Good sound insulation.
- Reflectors to direct sound evenly.

Doppler's effect:

The change in the apparent frequency of sound due to relative motion between the source and the observer is known as doppler's effect.

Ex: Change in ~~pit~~ the sound of a train as it passes by.